Review for Logarithmic Functions Unit Test

1. Rewrite each of the following in the form $\log_3$(expression).
   a. $\frac{1}{3}\log_3(x-2) - \log_3(x-1)$
   b. $-\log_3(5x^5) + \log_3(4x^4) - 1$
   c. $-2\log_3(x-3) + 3\log_3(x+5) - 2\log_3(x)$
   d. $\log_3(x) - 3\log_3(3x^5)$
   e. $-\log_3(5x^6) + 2\log_3(10x^5)$
   f. $2 - \log_3(3x^2) - 10\log_3(x)$

2. Which one of the following is equivalent to $\ln\left(\frac{A^2}{BC}\right)$
   (A) $2\ln A - (\ln B - \ln C)$
   (B) $2\ln A - \ln B - \ln C$
   (C) $2(\ln A - \ln B + \ln C)$
   (D) $2\ln A - \ln(B + C)$

3. Let $f(x) = 5^{-x}$ and let $g(x) = \log_5(2x)$. For $x > 0$, which of the following is an expression for $(f \circ g)(x)$?
   (A) $\frac{25}{x}$
   (B) $-x - 25$
   (C) $\frac{1}{2x}$
   (D) $-2x$

4. Let $f(x) = 2^{3x}$ and let $g(x) = \log_2(4x)$. Which one of the following is an expression for $(g \circ f)(x)$?
   (A) $x + 9$
   (B) $3x + 2$
   (C) $12x$
   (D) $64x$

5. The function M is given by $M(x) = \frac{3^x}{1+2^{kx}}$. Given that $M(2) = 1$, find $M(4)$.

6. The function g is given by $g(x) = 5 \cdot 8^x$. For what input value is 80 the output of g?

7. Find all values of x for which $\log_2(x^2 + 2x - 3) - \log_2(x - 1) = 3$.

8. Consider the functions f and g given by $f(x) = 2\log(x-2)$ and $g(x) = \log(x)$. In the xy-plane, what are all x-coordinates of the points of intersection of the graphs of f and g?

9. The function f is given by $f(x) = 3^x$. The graph of $g(x)$ is the image of the graph of $f(x)$ after a vertical translation -5 units and a horizontal translation -2 units. Find the x-intercept of the graph of $g(x)$.

10. Consider the functions g and h given by $g(x) = 9^x$ and $h(x) = 27^{x-1}$. In the xy-plane, what is the x-coordinate of the point of intersection of the graphs of g and h?

11. What are all values of x for which $\left(\frac{1}{2}\right)^{x+2} < 16$

12. What are all values of x for which $\log_2(x+3) + \log_2(x-4) \leq 3$

Let $f(x) = \log_4(x)$. Match each of the numbered descriptions on the left with the corresponding function on the right.

13. The graph of this function is the image of the graph of $f(x)$ after a vertical translation by -2 units.
14. The graph of this function is the image of the graph of $f(x)$ after horizontal dilation by a factor of $\frac{1}{4}$.
15. The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{3}$.
16. The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of 2.
17. The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{2}$.

$a(x) = \log_4(\sqrt[3]{x})$

$b(x) = \log_{16}(x)$

$c(x) = \log_2(x)$

$d(x) = \log_4(x) + 1$

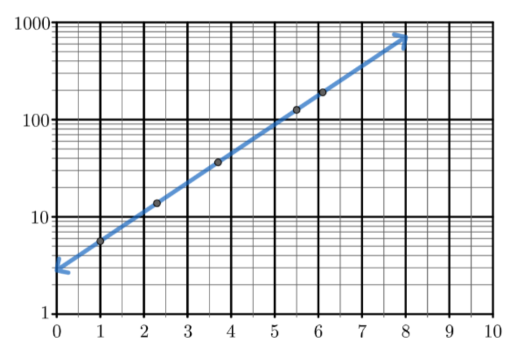
$e(x) = \log_4\left(\frac{x}{16}\right)$

18. The function f is given by $f(x) = \log_3(x)$. Find $f^{-1}(3)$.

19. If an exponential decay function is an appropriate model for a data set, then when that data set is plotted on a semi-log plot with the vertical axis scaled with a common logarithm, the distribution of the data will appear...
(A) Concave Down
(B) Concave Up
(C) Linear and Decreasing
(D) Linear with a Negative Initial Value

20. The function f has the property that for each time the input values double, the output values increase by 1. Which of the following could give the equation of $f(x)$? Select all that apply.
(A) $f(x) = \log_2(x) + 3$
(B) $f(x) = 3\log_2(x)$
(C) $f(x) = \log_2\left(\frac{x}{3}\right)$
(D) $f(x) = \log_2(x^3)$

21. A data set is represented using a semi-log plot where the vertical axis is logarithmically scaled with a base 10 logarithm. The data set appears linear on this semi-log plot. To the right, the data set and the linear pattern are shown on the semi-log plot. The data set is modeled well by the function $f(x)$. Which of the following is NOT true?
(A) $f(x)$ is an increasing function
(B) $f(x)$ is a concave up function
(C) $\lim_{x\to\infty} f(x) = \infty$
(D) $\lim_{x\to-\infty} f(x) = -\infty$



22. If $\lim_{x \to 4^-} g(x) = -\infty$ and $\lim_{x \to -\infty} g(x) = \infty$, which one of the following functions could be the equation of $g(x)$?
 (A) $g(x) = \ln(4 - x)$
 (B) $g(x) = \ln(x - 4)$
 (C) $g(x) = -\ln(4 - x)$
 (D) $g(x) = -\ln(x - 4)$

23. Find the domain of the function $f(x) = 2 + 3\log(3x + 10)$.

24. Which one of the following functions is decreasing and concave up over its domain?
 (A) $f(x) = 3\log_2(x - 1) - 4$
 (B) $g(x) = 3\log_2(-x + 1) - 4$
 (C) $h(x) = -3\log_2(-x + 1) + 4$
 (D) $k(x) = -3\log_2(x + 1) - 4$

25. Select values of $f(x)$ are given in the table below. Given that $f(x)$ is a logarithmic function, find the value of $f(97)$

x	-92	4	52	76	97
$f(x)$	-20	-15	-10	-5	$?$

26. (CALCULATOR/DESMOS) Consider the data set given below

x	0	1	2	3	4
$f(x)$	5.3	11.1	23.4	49.1	103.1

 a. This data set is modelled well by an exponential function of the form $f(x) = s(r)^x$ where s and r are positive constants. Use the regression capabilities of a graphing calculator to determine values of s and r.
 b. When this data set is plotted on a semi-log plot where the vertical axis is scaled using a natural logarithm, the distribution of points appears linear. Using your answer from (a), determine the apparent slope of the linear pattern.
 c. Using your answer from (a), determine the apparent initial value of the linear patten on the semi-log plot where the vertical axis is scaled using a natural logarithm.

27. (CALCULATOR/DESMOS) After a blizzard, the rain gutter starts to fall off Lauren's house. Every day, she measures the length of gutter that has detached from the wall. The table below shows the length (in feet) of gutter that has detached t days after the blizzard. A logarithmic function models this data well.

t	1	2	3	4	5
Length	7.7	8.5	9.1	9.7	10.1

 a. Use the logarithmic regression capabilities of a graphing calculator to predict the length of gutter detached from the wall on day $t = 10$.
 b. Predict on which day 13 feet of rain gutter will have detached from the wall. Round to the nearest day.

The free response portion of your test will consist of an FRQ 1 style problem and an FRQ 2 style problem. Like the AP exam, you will have 30 minutes to answer these two free response problems with the aid of a graphing calculator and Desmos.

28.

x	3	6	9	12	15
$f(x)$	48	24	12	6	3

Let f be a decreasing function defined for $x \geq 0$. The table gives values for $f(x)$ at selected values of x.

The function g is given by $g(x) = \frac{2x^3 - 11x + 1}{x - 5}$.

(A)

(i) The function h is defined by $h(x) = (g \circ f)(x) = g\big(f(x)\big)$. Find the value of $h(6)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find the value of $f^{-1}(3)$, or indicate that it is not defined.

(B)

(i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

(ii) Determine the end behavior of $g(x)$ as x decreases without bound. Express your answer using the mathematical notation of a limit.

(C)

(i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function.

(ii) Give a reason for your answer based on the relationship between the change in output values of f and the change in input values of f.

29. In 1975, Gary Dahl thought to himself “I bet I could make a lot of money selling pet rocks.” He began selling rocks packaged with a cardboard carrier (with ventilation holes of course) and straw bedding. One month after the launch of the Pet Rock ($t = 1$), they were selling at a rate of 4.7 thousand rocks per day. Three months later ($t = 4$), they were selling at a rate of 7.4 thousand rocks per day. But after one more month ($t = 5$), the popularity began to wane, and the rocks were only selling at a rate of 4 thousand rocks per day.

The sales rate of Pet Rocks can be modelled by the function S given by $S(t) = at^2 + bt + c$, where $S(t)$ is the sales rate (in rocks per day) t months after the launch of the product.

(A)

(i) Use the given data to write three equations that can be used to the find the values for constants a, b, and c in the expression for $S(t)$.

(ii) Find the values for a, b, and c as decimal approximations.

(B)

(i) Use the given data to find the average rate of change of change of the Pet Rock sales rate, in thousands per day per month, from $t = 1$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the Pet Rock sates rate for $t = 3$ months. Show the work that leads to your answer.

(iii) The average rate of change found in (i) can be used to estimate the Pet Rock sales rate for t-values in $1 < t < 4$. Will these estimates, found using the average rate of change, be less than or greater than the Pet Rock sales rate predicted by the model S? Your explanation should include a reference to the shape of the graph of S.

(C) Over the interval $5 \leq t \leq 6$, the sales rate of Pet Rocks increased due to the holiday season. Explain why the error in the model S increases over $5 \leq t \leq 6$.

Review for Logarithmic Functions Test Answers

1.
   a. $\log_3\left(\frac{\sqrt[3]{x-2}}{x-1}\right)$
   b. $\log_3\left(\frac{4}{15x}\right)$
   c. $\log_3\left(\frac{(x+5)^3}{x^2(x-3)^2}\right)$
   d. $\log_3\left(\frac{1}{27x^{14}}\right)$
   e. $\log_3(20x^4)$
   f. $\log_3\left(\frac{3}{x^{12}}\right)$
2. B ($2\ln A - \ln B - \ln C$)
3. C ($\frac{1}{2x}$)
4. B ($3x + 2$)
5. $\frac{81}{65}$
6. $x = \frac{4}{3}$
7. $x = 5$
8. $x = 4$
9. $(-2 + \log_3 5\,, 0)$
10. $x = 3$
11. $(-6, \infty)$
12. $(4,5]$
13. $e(x) = \log_4\left(\frac{x}{16}\right)$
14. $d(x) = \log_4(x) + 1$
15. $a(x) = \log_4(\sqrt[3]{x})$
16. $c(x) = \log_2(x)$
17. $b(x) = \log_{16}(x)$
18. $f^{-1}(3) = 27$
19. (C) Linear and Decreasing
20. A ($\log_2(x) + 3$) and C ($\log_2\left(\frac{x}{3}\right)$)
21. (D) $\lim_{x\to-\infty} f(x) = -\infty$
22. (A) $g(x) = \ln(4 - x)$
23. $\left(-\frac{10}{3}, \infty\right)$
24. (D) $k(x) = -3\log_2(x+1) - 4$
25. $f(97) = 10$

x	-92	4	52	76	88	94	97
$f(x)$	-20	-15	-10	-5	0	5	10

26.
   a. $s = 5.295$ and $r = 2.101$
   b. $\text{slope} = \ln(r) = 0.742$
   c. $\text{initial value} = \ln(s) = 1.667$
27.
   a. 11.025 feet
   b. $t = 37.609 \approx$ day 38

28.

(A)

i. $h(6) = g(f(6)) = g(24) = 1441.316$

ii. $f^{-1}(3) = 15$ since $f(15) = 3$

(B)

i. $x = -2.898$, $x = 1$, and $x = 1.898$

ii. $\lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} \frac{2x^3 - 11x + 1}{x - 5} = \infty$

(C)

i. Exponential

ii. Consecutive equally spaced input values correspond with output values that change proportionally ($r = \frac{1}{2}$).

29.

(A)

i. $4.7 = a(1)^2 + b(1) + c$
$7.4 = a(4)^2 + b(4) + c$
$4 = a(5)^2 + b(5) + c$

ii. $a = -1.075$, $b = 6.275$, $c = -0.5$

(B)

i. $\frac{S(4) - S(1)}{4 - 1} = \frac{7.4 - 4.7}{4 - 1} = \frac{2.7}{3} = 0.9$ thousand pet rocks per day per month
(or 900 pet rocks per day per month)

ii. $\frac{S(4) - S(3)}{4 - 3} \approx 0.9$
$\frac{7.4 - S(3)}{4 - 3} \approx 0.9$
$S(3) \approx 6.5$

iii. We are comparing outputs of the concave down function S to outputs of the secant line passing through S at $t = 1$ and $t = 4$. Since S is concave down, the graph of S is above the graph of the secant line between $t = 1$ and $t = 4$. Therefore, over this interval, the estimates found using the average rate of change from part (i) are less than the sales rates predicted by S.

(C) Error is the difference between the model and reality. At $t = 5$, the model S matches the actual sales rate of the Pet Rock. At this time, the error is zero. From $t = 5$ to $t = 6$, the model decreases but in reality sales increased. So, the error, the difference between the model and reality, increases over this interval.